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BRIEF COMMUNICATION

A METHOD FOR MEASUREMENT OF LOCAL SPECIFIC INTERFACIAL AREA

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1. INTRODUCTION

In the two-fluid models for two-phase flow systems the transport processes are governed by two sets of averaged conservation equations, each set representing the macroscopic balance of the mass, momentum and energy of a phase. The two sets of equations are coupled together through interaction terms which represent the transport of the mass, momentum and energy of each phase across the phase interface. In general, all interfacial transfer terms which appears in the two-fluid models can be exprsesed as the product of a driving force and the inverse of a length scale L_s at the interface (Ishii 1975):

$$\text{interfacial transfer term} = \text{driving force} \times \frac{1}{L_s}.$$

The driving forces are characterized by local transport mechanisms such as molecular and turbulent diffusion, whereas the term $1/L_s$, which represents the time-average of the interfacial area per unit volume (Kataoka *et al.* 1986) and is herein referred to as the local specific interfacial area, is related to the structure of the two-phase flow field. Knowledge of the local specific interfacial area is thus often required for a detailed analysis and prediction of the behavior of a two-phase flow system. This paper describes a method for measurement of local specific interfacial area using a quadruple-sensor electrical resistivity probe.

2. FORMULATION

According to Ishii (1975), the time-average of the specific interfacial area at a fixed position in space $\mathbf{x}_0$ is given by

$$\frac{1}{L_s} = \frac{1}{T} \sum_{j=1}^{N} \frac{1}{|\mathbf{v}_{ij} \cdot \mathbf{n}_j|}, \qquad [1]$$

where T is the length of the time interval over which the time averaging is considered, N is the number of times over the averaging period T that an interface passes through $\mathbf{x}_0$ and $\mathbf{v}_i$ and $\mathbf{n}$ are the velocity and outward-directed unit normal, respectively, of an interface at $\mathbf{x}_0$. The r.h.s. of [1] shall be related to measurable quantities.

Let $f(\mathbf{x}, t) = 0$ represent an interface. An event occurs at $\mathbf{x}_0$ when an interface passes through $\mathbf{x}_0$. The interface which pertains to the jth time an event occurs at $\mathbf{x}_0$ is referred to as the jth interface $f_j(\mathbf{x}_0, t_{oj}) = 0$. Assume that f_j is differentiable at $\mathbf{x} = \mathbf{x}_0$ and $t = t_{0j}$. Upon taking the material derivative of f_i at $\mathbf{x} = \mathbf{x}_0$ and $t = t_{0j}$, one finds that

$$\mathbf{v}_{ij}(\mathbf{x}_0, t_{0j}) \cdot \mathbf{n}_j(\mathbf{x}_0, t_{0j}) = -\frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|}. \qquad [2]$$

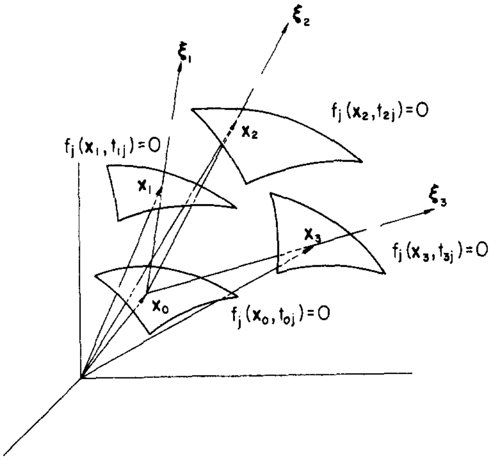

Figure 1

Substitution of [2] into [1] then gives

$$\frac{1}{L_s} = \frac{1}{T} \sum_{j=1}^{N} \left[\left| \frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|} \right| \right]^{-1}. \qquad [3]$$

Suppose now that the jth interface passes through three adjacent fixed points in space $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$, at times t_{1j}, t_{2j} and t_{3j}, respectively, as shown in figure 1; that is, $f_j(\mathbf{x}_k, t_{kj}) = 0$, $k = 1, 2, 3$. If the distances $s_k \equiv |\mathbf{x}_k - \mathbf{x}_0|$, $k = 1, 2, 3$, and the time differences $\Delta t_{kj} \equiv t_{kj} - t_{0j}$, $k = 1, 2, 3$, are small in comparison with the length scale and the time scale, respectively, of the system under consideration, then each of $f_j(\mathbf{x}_k, t_{kj})$, $k = 1, 2, 3$, can be written as a Taylor series expansion about $\mathbf{x} = \mathbf{x}_0$ and $t = t_{0j}$:

$$f_j(\mathbf{x}_k, t_{kj}) = f_j(\mathbf{x}_0, t_{0j}) + s_k \nabla f_j(\mathbf{x}_0, t_{0j}) \cdot \boldsymbol{\xi}_k + \Delta t_{kj} \frac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{oj})$$
$$+ \text{higher-order terms}, \quad k = 1, 2, 3, \qquad [4]$$

where $\nabla f_j(\mathbf{x}_0, t_{0j}) \cdot \boldsymbol{\xi}_k$ denotes the directional derivative of f_j in the direction of the unit vector $\boldsymbol{\xi}_k$ which is parallel to the line passing through $\mathbf{x}_0$ and $\mathbf{x}_k$. Neglecting the higher-order terms in [4] and making using of the fact that $f_j(\mathbf{x}_k, t_{kj}) = 0$, $k = 0, 1, 2, 3$, one obtains the following relation:

$$\frac{s_k}{\Delta t_{kj}} \approx - \frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{0j})}{\nabla f_j(\mathbf{x}_0, t_{0j}) \cdot \boldsymbol{\xi}_k}$$
$$= - \frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})| \mathbf{n}_j(\mathbf{x}_0, t_{0j}) \cdot \boldsymbol{\xi}_k}, \quad k = 1, 2, 3. \qquad [5]$$

In terms of the rectangular cartesian components ξ_{kx}, ξ_{ky} and ξ_{kz} of $\boldsymbol{\xi}_k$ and the direction cosines $\cos \alpha_j$, $\cos \beta_j$, and $\cos \gamma_j$ of $\mathbf{n}_j$, [5] can be rewritten as

$$\xi_{kx} \cos \alpha_j + \xi_{ky} \cos \beta_j + \xi_{kz} \cos \gamma_j = - \frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_0, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|} \frac{\Delta t_{kj}}{s_k}, \quad k = 1, 2, 3, \qquad [6]$$

which can then be solved, provided that the three unit vectors $\boldsymbol{\xi}_1$, $\boldsymbol{\xi}_2$ and $\boldsymbol{\xi}_3$ are linearly independent, to give

$$\cos\alpha_j = -\frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_o, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|}\frac{A_{1j}}{A_0}, \qquad [7]$$

$$\cos\beta_j = -\frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_o, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|}\frac{A_{2j}}{A_0} \qquad [8]$$

and

$$\cos\gamma_j = -\frac{\dfrac{\partial f_j}{\partial t}(\mathbf{x}_o, t_{0j})}{|\nabla f_j(\mathbf{x}_0, t_{0j})|}\frac{A_{3j}}{A_0}, \qquad [9]$$

where

$$A_0 = \begin{vmatrix} \xi_{1x} & \xi_{1y} & \xi_{1z} \\ \xi_{2x} & \xi_{2y} & \xi_{2z} \\ \xi_{3x} & \xi_{3y} & \xi_{3z} \end{vmatrix}, \qquad [10]$$

$$A_{1j} = \begin{vmatrix} \dfrac{\Delta t_{1j}}{s_1} & \xi_{1y} & \xi_{1z} \\ \dfrac{\Delta t_{2j}}{s_2} & \xi_{2y} & \xi_{2z} \\ \dfrac{\Delta t_{3j}}{s_3} & \xi_{3y} & \xi_{3y} \end{vmatrix}, \qquad [11]$$

$$A_{2j} = \begin{vmatrix} \xi_{1x} & \dfrac{\Delta t_{1j}}{s_1} & \xi_{1z} \\ \xi_{2x} & \dfrac{\Delta t_{2j}}{s_2} & \xi_{2z} \\ \xi_{3x} & \dfrac{\Delta t_{3j}}{s_3} & \xi_{3z} \end{vmatrix} \qquad [12]$$

and

$$A_{3j} = \begin{vmatrix} \xi_{1x} & \xi_{1y} & \dfrac{\Delta t_{1j}}{s_2} \\ \xi_{2x} & \xi_{2y} & \dfrac{\Delta t_{2j}}{s_2} \\ \xi_{3x} & \xi_{3y} & \dfrac{\Delta t_{3j}}{s_3} \end{vmatrix}. \qquad [13]$$

It follows from the identity $\cos^2\alpha_j + \cos^2\beta_j + \cos^2\gamma_j \equiv 1$ and [3] that

$$\frac{1}{L_s} = \frac{1}{T|A_0|}\sum_{j=1}^{N}\sqrt{A_{1j}^2 + A_{2j}^2 + A_{3j}^2}. \qquad [14]$$

Equation [14] indicates that $1/L_s$ can be unambiguously determined from three sets of measurable quantities Δt_{1j}, Δt_{2j} and Δt_{3j}, $j = 1, \ldots, N$, provided that the passage of interfaces through fixed locations in a two-phase flow field can be detected experimentally. Recall that only two assumptions have been made in arriving at [14]. The two assumptions are: (1) f_j is differentiable at $\mathbf{x} = \mathbf{x}_0$ and $t = t_{0j}$; and (2) s_k and Δt_{kj}, $k = 1, 2, 3$, are small enough to allow the higher-order terms in [4] to be neglected. Inasmuch as the shape of the interfaces has no bearing upon the derivation of [14], the range of applicability of [14] covers *all* two-phase flow patterns. For the particular case of vertical flow of bubbly mixtures consisting of spherical bubbles it is possible to determine $1/L_s$ from a single set of measurement of Δt_{kj} alone (Tan & Ishii 1989).

3. MEASUREMENT METHOD

The detection of the passage of interfaces through fixed locations in a two-phase flow field can be accomplished with probe techniques (Jones & Delhaye 1976), which are based on the fact that certain optical and electrical properties of fluids can be measured by miniature sensors. For the purpose of illustration, the electrical resistivity probe technique is considered in the following. This technique, which was first proposed by Neal & Bankoff (1963) for determination of bubble parameters in gas–liquid bubbly flows, consists of the continuous measurement of electricial resistivity in the two-phase stream by means of a sensor which is the exposed tip of an otherwise electrically insulated metal wire. Typical signals from an electrical resistivity probe show deviations from the ideal two-state square wave signals (Park *et al.* 1969); this deviation is to a large measure due to the deformation of the interface before the sensor enters from one phase into the other phase (Jones & Delhaye 1976). The signals are generally transformed into two-state square waves with the help of an on-line Schmidt trigger and then passed to other instruments for further on-line or off-line analysis (Werther 1974; Hoffer & Resnick 1975; Serizawa *et al.* 1975; Herringe & Davis 1976; Veteau 1981). In this approach to data analysis the threshold voltage for the Schmidt trigger is determined *beforehand* through comparison of the void fraction thus obtained, which is a function of the threshold voltage, with that measured with other techniques. The transformation of a signal from its original form into that of a two-state square wave is irrevocable. It is also doubtful that a meaningful estimate of the additional experimental uncertainty due to electro-chemical phenomena on the sensor can be made. Hence, an alternative approach in which the original signal is digitized and stored in an on-line data acquisition microcomputer, and the selection of the threshold voltage is accomplished through software, is considered herein. The algorithm for selecting the threshold voltage is described as follows.

Consider a gas–liquid mixture flowing upwards in a vertical test section made of circular pipes. A quadruple-sensor electrical resistivity probe is made to traverse along the diameter of the cross section. The locations of the tips of the four sensors are identified with the four fixed locations $\mathbf{x}_k$, $k = 0, 1, 2, 3$, considered in section 2. In terms of cylindrical coordinates they are represented as $\mathbf{x}_k = (z_k, r_k^{(m)})$, $k = 0, 1, 2, 3$, where $r^{(m)}$ denotes a fixed radial coordinate which corresponds to the mth traversing stop of the probe. Suppose that there are a total of M stops and that the sampling period at each stop remains constant at T for an experimental run. Let $V_k^{(m)}(t)$ denote the time-history records of signals from the four sensors at the mth stop, i.e.

$$V_k^{(m)}(t) \equiv V_k(z_k, r_k^{(m)}, t), \quad t^{(m)} - \frac{T}{2} \leqslant t \leqslant t^{(m)} + \frac{T}{2},$$

and let V_{kT} denote threshold voltages which apply to $V_k^{(m)}$, $m = 1, \ldots, M$.

As shown in figure 2, the gas-contact period of a sensor over the sampling period T is a function of the threshold voltage associated with that sensor. Consequently, the local void fractions $\epsilon_k^{(m)}$, which are defined as

$$\begin{aligned} \epsilon_k^{(m)} &\equiv \epsilon_k(z_k, r_k^{(m)}, t^{(m)}) \\ &\overset{\text{def}}{=} \frac{1}{T} \sum_{j=1,\, j\,\text{odd}}^{N-1} (t_{k,j+1}^{(m)} - t_{kj}^{(m)}), \quad k = 0, 1, 2, 3, \end{aligned} \qquad [15]$$

are functions of the threshold voltages $V_{kT}, k = 0, 1, 2, 3$, respectively. Furthermore, when the process is ergodic, which is assumed to be the case, $\epsilon_k^{(m)}$ do not depend on $t^{(m)}$ so they can be averaged over the radial position to give line-averaged void fractions

$$\langle \epsilon_k \rangle_l \approx \frac{1}{2R} \sum_{m=1}^{M-1} (r_k^{(m+1)} - r_k^{(m)})(\epsilon_k^{(m+1)} + \epsilon_k^{(m)}), \quad k = 0, 1, 2, 3. \qquad [16]$$

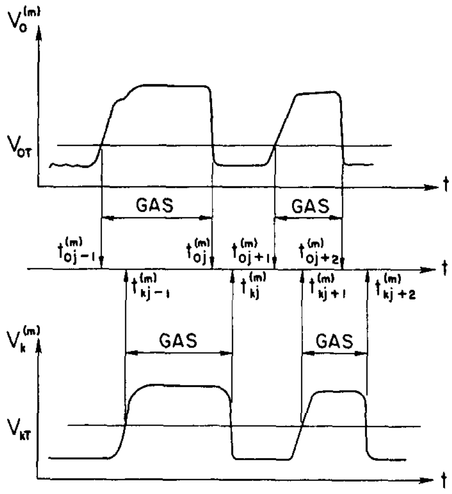

Figure 2

The line-averaged void fractions $\langle\epsilon_k\rangle_l$ thus obtained are also functions of the threshold voltages V_{kT}, $k = 0, 1, 2, 3$, respectively. The threshold voltages V_{kT} are adjusted until agreement is reached between $\langle\epsilon_k\rangle_l$ and the line-averaged void fractions $\langle\epsilon_k\rangle_{\text{expt}}$ obtained directly (and concurrently) by means of one of the techniques for measurement of line void fraction. Specifically, the method of adjustment consists of an iterative scheme for approximating the roots of the nonlinear equations:

$$F_k(V_{kT}) \stackrel{\text{def}}{=} \langle\epsilon_k\rangle_l(V_{kT}) - \langle\epsilon_k\rangle_{\text{expt}} = 0, \quad k = 0, 1, 2, 3. \qquad [17]$$

In regions where the flow is fully developed, the volume-averaged void fraction is identical to the line-averaged ones; the adjustment of V_{kT}, $k = 0, 1, 2, 3$, can be based on the comparison between the volume-averaged void fraction determined from concurrent differential pressure measurements and $\langle\epsilon_k\rangle_l$, $k = 0, 1, 2, 3$, respectively. Once V_{kT} are determined, t_{kj}, $j = 1, \ldots, N$, are determined and the local specific interfacial area can be calculated with the help of [14].

Note that the practicability of [14] depends on the experimental capability of measuring Δt_{kj}. As Δt_{kj} are functions of the threshold voltages V_{kT}, the resistivity probe technique is not a stand-alone one for measurement of local specific interfacial area. The accuracy of the local specific interfacial area thus measured is necessarily, at best, as good as that of the void fraction measurement.

It is worthwhile at this point to make some remarks about the conditions under which the assumption that s_k and Δt_{kj}, $k = 1, 2, 3$, are small in comparison with the length scale and the time scale, respectively, of the physical system under consideration can be justified. In accordance with the basic concept of the two-fluid model, a physical dimension which characterizes the degree of dispersion or degree of separation, e.g. a typical bubble diameter in the case of bubbly flow or a typical liquid film thickness in the case of annular flow, can be considered as the length scale and the time scale can be regarded as a measure of the time it takes for the two-phase mixture to travel a distance equal to the length scale. Thus, a bubbly flow with bubbles with diameters of the order of 1 mm entails a probe in which the distances between the tips of sensors are of the order of 0.1 mm. When the distances are fixed to be of the order of 1 mm, the diameters of bubbles should be of the order of 1 cm for the probe techniques for measurement of local specific interfacial area to be applicable.

It should also be emphasized that the resistivity probe technique is an intrusive one. The first sensor could slow down the interfaces, thereby resulting in longer Δt_{kj} and therefore larger $1/L_s$. In the case of bubbly flow, the probe could detour smaller bubbles, thereby giving smaller $1/L_s$. This latter effect on measured $1/L_s$ is very dependent on the bubble size distribution.

4. SUMMARY

A mathematical relation between local specific interfacial area $1/L_s$ and measurable quantities is derived based on kinematics and geometry alone. The relation indicates that $1/L_s$ can be unambiguously measured provided that the passage of interfaces through fixed locations in the two-phase field can be detected experimentally. A quadruple-sensor electrical resistivity probe technique for measurement of $1/L_s$ is described. Limitations of the technique are briefly discussed.

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